Multidimensional cosmological and spherically symmetric solutions with intersecting p-branes

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Abstract. Multidimensional model describing the "cosmological" and/ or spherically symmetric configuration with $(n + 1)$ Einstein spaces in the theory with several scalar fields and forms is considered. When electro-magnetic composite p-brane ansatz is adopted, n "internal" spaces are Ricci-flat, one space M_0 has a non-zero curvature, and all p-branes do not "live" in M_0, a class of exact solutions is obtained if certain block-orthogonality relations on p-brane vectors are imposed. A subclass of spherically-symmetric solutions containing non-extremal p-brane black holes is considered. Post-Newtonian parameters are calculated and some examples are considered.

Keywords. *P*-branes, multidimensional gravity, black holes.

1 Introduction

The necessity of studying multidimensional models of gravitation [1, 2] is motivated by several reasons. First, the main trend of modern physics is the unification of all known fundamental physical interactions: electromagnetic, weak, strong and gravitational ones. During last decades there was a significant progress in unifying weak and electromagnetic interactions, some more modest achievements in GUT, supersymmetric, string and superstring theories.

Now theories with membranes, p-branes and more vague M- and F-theories [4, 6, 7, 8] are being created and studied. Having no any definite successful theory of unification now, it is desirable to study the common features of these theories and their applications to solving basic problems of modern gravity.

Second, multidimensional gravitational models, as well as scalar-tensor theories of gravity, are the theoretical framework for describing possible temporal and range variations of fundamental physical constants [3]. These ideas originated from earlier papers of P.Dirac (1937) on relations between fenomena of micro and macro worlds and up till now they are under a thorough study both theoretically and experimentally.

Bearing in mind that multidimensional gravitational models are certain generalizations of general relativity which is tested reliably for weak fields up to 0,001 (they may be viewed as some effective scalar-tensor theories in simple variants in four dimensions) it is quite natural to inquire about their possible observational or experimental windows. ¿From what we already know, among these windows are :

– possible deviations from the Newton and Coulomb laws,

– possible variations of the effective gravitational constant with a time rate less than the Hubble one,

– possible existence of monopole modes in gravitational waves,

– different behaviour of strong field objects, such as multidimensional black holes, wormholes and p-branes,

– standard cosmological tests etc.

As no accepted unified model exists, in our approach we adopt simple, but general from the point of view of number of dimensions, models based on multidimensional Einstein equations with or without sources of different nature:

– cosmological constant,

– perfect and viscous fluids,

– scalar and electromagnetic fields,

– plus their interactions,

– fields of antisymmetric forms (related to p-branes) etc.

Our main objective was and is to obtain exact solutions (integrable models) for these model self-consistent systems and then to analyze them in cosmological, spherically and axially symmetric cases. In our view this is a natural and most reliable way to study highly nonlinear systems. It is done mainly within the Riemannian geometry. Some simple models in integrable Weyle geometry and with torsion were studied also.

1.1 Problem of Stability of G

1.1.1 Absolute G measurements

The value of the Newton's gravitational constant G as adopted by CODATA in 1986 is based on Luther and Towler measurements of 1982.

Even at that time other existing on 100ppm level measurements deviated ¿from this value more than their uncertainties [12]. During last years the situation, after very precise measurements of G in Germany and New Zealand, became much more vague. Their results deviate from the official CODATA value from 600 ppm at minimal to 630 ppm at maximal values.

As it is seen from the most recent data announced in November 1998 at the Cavendish conference in London the situation with terrestrail absolute G measurements is not improving. The reported values for G (in units of 10^{-11}) and their estimated error in ppm are as follows:

Fitzgerald and Armstrong	6.6742	90 ppm
	6.6746	134
Nolting et al. (Zurich)	6.6749	210
Meyer et al. (Wuppethal)	6.6735	240
Karagioz et al. (Moscow)	6.6729	75
Richman et al.	6.683	1700
Schwarz et al.	6.6873	1400
CODATA (1986, Luther)	6.67259	128

This means that either the limit of terrestrial accuracies is reached or we have some new physics entering the measurement procedure [13, 14]. First means that we should shift to space experiments to measure G [15] and second means that more thorough study of theories generalizing Einstein's general relativity is necessary.

1.1.2 Data on temporal variations of G

Dirac's prediction based on his Large Numbers Hypothesis is $\dot{G}/G = (-5)10^{-11}$ $year^{-1}$. Other hypotheses and theories, in particular some scalar-tensor or multidimensional ones, predict these variations on the level of $10^{-12}-10^{-13}$ per year. As to experimental or observational data, the results are rather nonconclusive. The most reliable ones are based on Mars orbiters and landers (Hellings,1983) and on lunar laser ranging (Muller et al., 1993; Williams et al., 1996). They are not better than 10^{-12} per year [16]. Here once more we see that there is a need for corresponding theoretical and experimental studies. Probably, future space missions to other planets will be a decisive step in solving the problem of temporal variations of G and defining the fates of different theories which predict them as the larger is the time interval between successive measurements and, of course, the more precise they are, the more stringent results will be obtained.

1.1.3 Nonnewtonian interactions (EP and ISL tests)

Nearly all modified theories of gravity and unified theories predict also some deviations from the Newton law (ISL) or composite-dependant violation of the Equivalence Principle (EP) due to an appearance of new possible massive particles (partners) [11]. Experimental data exclude the existence of these particles nearly at all ranges except less than millimeter and also at meters and hundreds of meters ranges. The most recent result in the range of 20-500 m was obtained by Achilli et al [17]. They found the positive result for the deviation from the Newton law with the Yukawa potential strength alpha between 0,13 and 0,25. Of course, these results need to be verified in other independent experiments, probably in space ones.

1.2 Multidimensional Models

The history of multidimensional approach starts from the well-known papers of T.K. Kaluza and O. Klein [18, 19] on 5-dimensional theories which opened an interest (see [20, 21, 22, 23]) to investigations in multidimensional gravity. These ideas were continued by P.Jordan [24] who suggested to consider the more general case $g_{55} \neq$ const leading to the theory with an additional scalar field. The papers [18, 19, 24] were in some sense a source of inspiration for C. Brans and R.H. Dicke in their well-known work on the scalar-tensor gravitational theory [25]. After their work a lot of investigations were done using material or fundamental scalar fields, both conformal and nonconformal (see details in [3]).

The revival of ideas of many dimensions started in 70th and continues now. It is due completely to the development of unified theories. In the 70th an interest to multidimensional gravitational models was stimulated mainly by: i) the ideas of gauge theories leading to the non-Abelian generalization of Kaluza-Klein approach and by ii) supergravitational theories [26, 27]. In the 80th the supergravitational theories were "replaced" by superstring models [28]. Now it is heated by expectations connected with overall M-theory or even some F-theory. In all these theories 4-dimensional gravitational models with extra fields were obtained from some multidimensional model by a dimensional reduction based on the decomposition of the manifold

$$M = M^4 \times M_{int},$$

where M^4 is a 4-dimensional manifold and M_{int} is some internal manifold (mostly considered as a compact one).

The earlier papers on multidimensional cosmology dealt with multidimensional Einstein equations and with a block-diagonal cosmological metric defined on the manifold $M = \mathbb{R} \times M_0 \times ... \times M_n$ of the form

$$g = -dt \otimes dt + \sum_{r=0}^{n} a_r^2(t) g^r$$

where (M_r, g^r) are Einstein spaces, $r = 0, \ldots, n$ [30]–[62]. In some of them a cosmological constant and simple scalar fields were used also [104].

In [40, 41, 45, 50, 51, 57, 69, 70] the models with higher dimensional "perfect-fluid" were considered. In these models pressures (for any component) are proportional to the density

$$p_r = \left(1 - \frac{u_r}{d_r}\right) \rho,$$

$r = 0, \ldots, n$, where d_r is a dimension of M_r. Such models are reduced to pseudo-Euclidean Toda-like systems with the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{x}^i \dot{x}^j - \sum_{k=1}^{m} A_k e^{u_i^k x^i}$$

and the zero-energy constraint $E = 0$. In a classical case exact solutions with Ricci-flat (M_r, g^r) for 1-component case were considered by many authors (see, for example, [38, 39, 50, 51, 69, 70, 97] and references therein). For the two component perfect-fluid there were solutions with two curvatures, i.e. $n = 2$, when $(d_1, d_2) = (2, 8), (3, 6), (5, 5)$ [106] and corresponding non-singular solutions from [141]. Among the solutions [106] there exists a special class of Milne-type solutions. Recently some interesting extensions of 2-component solutions were obtained in [107].

It should be noted that the pseudo-Euclidean Toda-like systems are not well-studied yet. There exists a special class of equations of state that gives rise to the Euclidean Toda models. First such solution was considered in [70] for the Lie

algebra a_2. Recently the case of $a_n = sl(n+1)$ Lie algebras was considered and the solutions were expressed in terms of a new elegant representation (obtained by Anderson) [105].

The cosmological solutions may have regimes with: i) spontaneous and dynamical compactifications; ii) Kasner-like and billiard behavior near the singularity; iii) inflation and izotropization for large times (see, for example, [104, 69]).

Near the singularity one can have an oscillating behavior like in the well-known mixmaster (Bianchi-IX) model. Multidimensional generalizations of this model were considered by many authors (see, for example, [29, 76, 77, 78]). In [79, 80, 81] the billiard representation for multidimensional cosmological models near the singularity was considered and the criterion for the volume of the billiard to be finite was established in terms of illumination of the unit sphere by point-like sources. For perfect-fluid this was considered in detail in [81]. Some interesting topics related to general (non-homogeneous) situation were considered in [82].

Multidimensional cosmological models have a generalization to the case when the bulk and shear viscosity of the "fluid" is taken into account [108]. Some classes of exact solutions were obtained, in particular nonsingular cosmological solutions, generation of mass and entropy in the Universe.

Multidimensional quantum cosmology based on the Wheeler-DeWitt (WDW) equation

$$\hat{H}\Psi = 0,$$

where Ψ is the so-called "wave function of the universe", was treated first in [52], (see also [64]). This equation was considered for the vacuum case in [52] and integrated in a very special situation of 2-spaces. The WDW equation for the cosmological constant and for the "perfect-fluid" was investigated in [68, 104] and [69] respectively.

Exact solutions in 1-component case were considered in detail in [97] (for perfect fluid). In [60] the multidimensional quantum wormholes were suggested, i.e. solutions with a special-type behavior of the wave function (see [65]).

These solutions were generalized to account for cosmological constant in [68, 104] and to the perfect-fluid case in [69, 97]. In [81] the "quantum billiard" was obtained for multidimensional WDW solutions near the singularity. It should be also noted that the "third-quantized" multidimensional cosmological models were considered in several papers [59, 103, 97]. One may point out that in all cases when we had classical cosmological solutions in many dimensions, the corresponding quantum cosmological solutions were found also.

Cosmological solutions are closely related to solutions with the spherical symmetry. Moreover, the scheme of obtaining them is very similar to the cosmological approach. The first multidimensional generalization of such type was considered by D. Kramer [87] and rediscovered by A.I. Legkii [88], D.J. Gross and M.J. Perry [89] (and also by Davidson and Owen). In [91] the Schwarzschild solution was generalized to the case of n internal Ricci-flat spaces and it was shown that black hole configuration takes place when scale factors of internal spaces are constants. In [92] an analogous generalization of the Tangherlini solution

[90] was obtained. These solutions were also generalized to the electrovacuum case [93, 96, 94]. In [95, 94] multidimensional dilatonic black holes were singled out. An interesting theorem was proved in [94] that "cuts" all non-black-hole configurations as non-stable under even monopole perturbations. In [98] the extremely-charged dilatonic black hole solution was generalized to multicenter (Majumdar-Papapetrou) case when the cosmological constant is non-zero.

We note that for $D = 4$ the pioneering Majumdar-Papapetrou solutions with conformal scalar field and electromagnetic field were considered in [161].

At present there exists a special interest to the so-called M- and F-theories etc. [4, 6, 7, 8]. These theories are "supermembrane" analogues of superstring models [28] in $D = 11, 12$ etc. The low-energy limit of these theories leads to models governed by the Lagrangian

$$\mathcal{L} = R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{\theta_a}{n_a!} \exp[2\lambda_a(\varphi)](F^a)^2,$$

where g is metric, $F^a = dA^a$ are forms of rank $F^a = n_a$, and φ^α are scalar fields.

In [143] it was shown that after dimensional reduction on the manifold $M_0 \times M_1 \times \ldots \times M_n$ and when the composite p-brane ansatz is considered the problem is reduced to the gravitating self-interacting σ-model with certain constraints imposed. For electric p-branes see also [141, 142, 144] (in [144] the composite electric case was considered). This representation may be considered as a powerful tool for obtaining different solutions with intersecting p-branes (analogs of membranes). In [143, 168] the Majumdar-Papapetrou type solutions were obtained (for non-composite electric case see [141, 142] and for composite electric case see [144]). These solutions correspond to Ricci-flat (M_i, g^i), $i = 1, \ldots, n$, and were generalized also to the case of Einstein internal spaces [143]. Earlier some special classes of these solutions were considered in [126, 127, 128, 146, 147, 148]. The obtained solutions take place, when certain orthogonality relations (on couplings parameters, dimensions of "branes", total dimension) are imposed. In this situation a class of cosmological and spherically-symmetric solutions was obtained [166]. Special cases were also considered in [130, 153, 154, 152]. The solutions with the horizon were considered in details in [131, 149, 150, 151, 166]. In [151, 167] some propositions related to i) interconnection between the Hawking temperature and the singularity behaviour, and ii) to multitemporal configurations were proved.

It should be noted that multidimensional and multitemporal generalizations of the Schwarzschild and Tangherlini solutions were considered in [96, 164], where the generalized Newton's formulas in multitemporal case were obtained.

We note also that there exists a large variety of Toda solutions (open or closed) when certain intersection rules are satisfied [166].

In [166] (see also [158]) the Wheeler-DeWitt equation was integrated for intersecting p-branes in orthogonal case and corresponding classical solutions were obtained also. A slightly different approach was suggested in [155]. (For non-composite case see also [154].)

In [159, 169] exact solutions for multidimensional models with intersecting

p-branes in case of static internal spaces were obtained. They turned to be de Sitter or anti- de Sitter type. Generation of the effective cosmological constant and inflation via p-branes was demonstrated there. These solutions may be considered as an interesting first step for a quantum description of low-energy limits in different super-p-branes theories.

In this paper we continue our investigations of p-brane solutions (see for example [112, 113, 148] and references therein) based on sigma-model approach [143, 142, 144] . (For pure gravitational sector see [109, 141].)

Here we consider a cosmological and/or spherically symmetric case , when all functions depend upon one variable (time or radial variable). The model under consideration contains several scalar fields and antisymmetric forms and is governed by action (2.1).

The considered cosmological model contains some stringy cosmological models (see for example [156]). It may be obtained (at classical level) from multidimensional cosmological model with perfect fluid [69, 70] as a special case.

Here we find a family of solutions depending on one variable describing the (cosmological or spherically symmetric) "evolution" of $(n+1)$ Einstein spaces in the theory with several scalar fields and forms. When an electro-magnetic composite p-brane ansatz is adopted the field equations are reduced to the equations for Toda-like system.

In the case when n "internal" spaces are Ricci-flat, one space M_0 has a non-zero curvature, and all p-branes do not "live" in M_0, we find a family of solutions (Section 4) to the equations of motion (equivalent to equations for Toda-like Lagrangian with zero-energy constraint [166]) if certain block-orthogonality relations on p-brane vectors U^s are imposed. These solutions generalize the solutions from [166] with orthogonal set of vectors U^s. A special class of "block-orthogonal" solutions (with coinciding parameters ν_s inside blocks) was considered earlier in [167].

Here we consider a subclass of spherically-symmetric solutions (Sect. 5). This subclass contains non-extremal p-brane black holes for zero values of "Kasner-like" parameters. The relation for the Hawking temperature is presented (in the black hole case).

We also calculate Post-Newtonian parameters β and γ (Eddington parameters) for the spherically-symmetric solutions (Sect. 6). These parameters may be useful for possible physical applications.

2 The model

Here like in [143] we consider the model governed by the action

$$
\begin{aligned}
S = \quad & \frac{1}{2\kappa^2} \int_M d^D z \sqrt{|g|} \{ R[g] - 2\Lambda - h_{\alpha\beta} \, g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta \qquad (2.1) \\
& - \sum_{a \in \Delta} \frac{\theta_a}{n_a!} \exp[2\lambda_a(\varphi)](F^a)_g^2 \} + S_{GH},
\end{aligned}
$$

where $g = g_{MN}dz^M \otimes dz^N$ is the metric $(M, N = 1, \ldots, D)$, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector from dilatonic scalar fields, $(h_{\alpha\beta})$ is a non-degenerate symmetric $l \times l$ matrix $(l \in \mathbb{N})$, $\theta_a = \pm 1$,

$$F^a = dA^a = \frac{1}{n_a!} F^a_{M_1 \ldots M_{n_a}} dz^{M_1} \wedge \ldots \wedge dz^{M_{n_a}} \qquad (2.2)$$

is a n_a-form $(n_a \geq 1)$ on a D-dimensional manifold M, Λ is cosmological constant and λ_a is a 1-form on $\mathbb{R}^l$: $\lambda_a(\varphi) = \lambda_{a\alpha}\varphi^\alpha$, $a \in \Delta$, $\alpha = 1, \ldots, l$. In (2.1) we denote $|g| = |\det(g_{MN})|$,

$$(F^a)^2_g = F^a_{M_1 \ldots M_{n_a}} F^a_{N_1 \ldots N_{n_a}} g^{M_1 N_1} \ldots g^{M_{n_a} N_{n_a}}, \qquad (2.3)$$

$a \in \Delta$, where Δ is some finite set, and S_{GH} is the standard Gibbons-Hawking boundary term [163]. In the models with one time all $\theta_a = 1$ when the signature of the metric is $(-1, +1, \ldots, +1)$.

The equations of motion corresponding to (2.1) have the following form

$$R_{MN} - \frac{1}{2}g_{MN}R = T_{MN} - \Lambda g_{MN}, \qquad (2.4)$$

$$\triangle[g]\varphi^\alpha - \sum_{a \in \Delta} \theta_a \frac{\lambda_a^\alpha}{n_a!} e^{2\lambda_a(\varphi)}(F^a)^2_g = 0, \qquad (2.5)$$

$$\nabla_{M_1}[g](e^{2\lambda_a(\varphi)} F^{a, M_1 \ldots M_{n_a}}) = 0, \qquad (2.6)$$

$a \in \Delta$; $\alpha = 1, \ldots, l$. In (2.5) $\lambda_a^\alpha = h^{\alpha\beta}\lambda_{a\beta}$, where $(h^{\alpha\beta})$ is matrix inverse to $(h_{\alpha\beta})$. In (2.4)

$$T_{MN} = T_{MN}[\varphi, g] + \sum_{a \in \Delta} \theta_a e^{2\lambda_a(\varphi)} T_{MN}[F^a, g], \qquad (2.7)$$

where

$$T_{MN}[\varphi, g] = h_{\alpha\beta}\left(\partial_M\varphi^\alpha \partial_N\varphi^\beta - \frac{1}{2}g_{MN}\partial_P\varphi^\alpha \partial^P\varphi^\beta\right), \qquad (2.8)$$

$$T_{MN}[F^a, g] = \frac{1}{n_a!}\left[-\frac{1}{2}g_{MN}(F^a)^2_g + n_a F^a_{MM_2 \ldots M_{n_a}} F^{a, M_2 \ldots M_{n_a}}_N\right]. \qquad (2.9)$$

In (2.5), (2.6) $\triangle[g]$ and $\nabla[g]$ are Laplace-Beltrami and covariant derivative operators respectively corresponding to g.

Let us consider the manifold

$$M = \mathbb{R} \times M_0 \times \ldots \times M_n \qquad (2.10)$$

with the metric

$$g = we^{2\gamma(u)}du \otimes du + \sum_{i=0}^n e^{2\phi^i(u)}g^i, \qquad (2.11)$$

where $w = \pm 1$, u is a distinguished coordinate which, by convention, will be called "time"; $g^i = g^i_{m_i n_i}(y_i) dy_i^{m_i} \otimes dy_i^{n_i}$ is a metric on M_i satisfying the equation

$$R_{m_i n_i}[g^i] = \xi_i g^i_{m_i n_i}, \tag{2.12}$$

$m_i, n_i = 1, \ldots, d_i$; $d_i = \dim M_i$, $\xi_i = \text{const}$, $i = 0, \ldots, n$; $n \in \mathbb{N}$. Thus, (M_i, g^i) are Einstein spaces. The functions $\gamma, \phi^i \colon (u_-, u_+) \to \mathbb{R}$ are smooth.

Each manifold M_i is assumed to be oriented and connected, $i = 0, \ldots, n$. Then the volume d_i-form

$$\tau_i = \sqrt{|g^i(y_i)|}\; dy_i^1 \wedge \ldots \wedge dy_i^{d_i}, \tag{2.13}$$

and the signature parameter

$$\varepsilon(i) = \text{sign}\det(g^i_{m_i n_i}) = \pm 1 \tag{2.14}$$

are correctly defined for all $i = 0, \ldots, n$.

Let

$$\Omega_0 = \{\emptyset, \{0\}, \{1\}, \ldots, \{n\}, \{0,1\}, \ldots, \{0,1,\ldots,n\}\} \tag{2.15}$$

be a set of all subsets of

$$I_0 \equiv \{0, \ldots, n\}. \tag{2.16}$$

Let $I = \{i_1, \ldots, i_k\} \in \Omega_0$, $i_1 < \ldots < i_k$. We define a form

$$\tau(I) \equiv \tau_{i_1} \wedge \ldots \wedge \tau_{i_k}, \tag{2.17}$$

of rank

$$d(I) \equiv \sum_{i \in I} d_i, \tag{2.18}$$

and a corresponding p-brane submanifold

$$M_I \equiv M_{i_1} \times \ldots \times M_{i_k}, \tag{2.19}$$

where $p = d(I) - 1$ ($\dim M_I = d(I)$). We also define ε-symbol

$$\varepsilon(I) \equiv \varepsilon(i_1) \ldots \varepsilon(i_k). \tag{2.20}$$

For $I = \emptyset$ we put $\tau(\emptyset) = \varepsilon(\emptyset) = 1$, $d(\emptyset) = 0$.

For fields of forms we adopt the following "composite electro-magnetic" ansatz

$$F^a = \sum_{I \in \Omega_{a,e}} \mathcal{F}^{(a,e,I)} + \sum_{J \in \Omega_{a,m}} \mathcal{F}^{(a,m,J)}, \tag{2.21}$$

where

$$\mathcal{F}^{(a,e,I)} = d\Phi^{(a,e,I)} \wedge \tau(I), \tag{2.22}$$

$$\mathcal{F}^{(a,m,J)} = e^{-2\lambda_a(\varphi)} * \left(d\Phi^{(a,m,J)} \wedge \tau(J) \right), \tag{2.23}$$

$a \in \Delta$, $I \in \Omega_{a,e}$, $J \in \Omega_{a,m}$ and

$$\Omega_{a,e}, \Omega_{a,m} \subset \Omega_0. \tag{2.24}$$

(For empty $\Omega_{a,v} = \emptyset$, $v = e, m$, we put $\sum_{\emptyset} = 0$ in (2.21)). In (2.23) $* = *[g]$ is the Hodge operator on (M, g).

For the potentials in (2.22), (2.23) we put

$$\Phi^s = \Phi^s(u), \tag{2.25}$$

$s \in S$, where

$$S = S_e \sqcup S_m, \qquad S_v \equiv \sqcup_{a \in \Delta} \{a\} \times \{v\} \times \Omega_{a,v}, \tag{2.26}$$

$v = e, m$. Here $\sqcup$ means the union of non-intersecting sets. The set S consists of elements $s = (a_s, v_s, I_s)$, where $a_s \in \Delta$, $v_s = e, m$ and $I_s \in \Omega_{a,v_s}$ are "color", "electro-magnetic" and "brane" indices, respectively.

For dilatonic scalar fields we put

$$\varphi^\alpha = \varphi^\alpha(u), \tag{2.27}$$

$\alpha = 1, \ldots, l$.

¿From (2.22) and (2.23) we obtain the relations between dimensions of p-brane worldsheets and ranks of forms

$$d(I) = n_a - 1, \quad I \in \Omega_{a,e}, \tag{2.28}$$
$$d(J) = D - n_a - 1, \quad J \in \Omega_{a,m}, \tag{2.29}$$

in electric and magnetic cases respectively.

3 σ-model representation

Here, like in [166], we impose a restriction on p-brane configurations, or, equivalently, on $\Omega_{a,v}$. We assume that the energy momentum tensor (T_{MN}) has a block-diagonal structure (as it takes place for (g_{MN})). Sufficient restrictions on $\Omega_{a,v}$ that guarantee a block-diagonality of (T_{MN}) are presented in Appendix 1.

It follows from [143] (see Proposition 2 in [143]) that the equations of motion (2.4)–(2.6) and the Bianchi identities

$$d\mathcal{F}^s = 0, \quad s \in S \tag{3.1}$$

for the field configuration (2.11), (2.21)–(2.23), 2.25, (2.27) with the restrictions (8.2), (8.3) (from Appendix 1) imposed are equivalent to equations of motion for σ-model with the action

$$S_\sigma = \frac{\mu_*}{2} \int du \mathcal{N} \left\{ G_{ij} \dot{\phi}^i \dot{\phi}^j + h_{\alpha\beta} \dot{\varphi}^\alpha \dot{\varphi}^\beta \right. \tag{3.2}$$

$$\left. + \sum_{s \in S} \varepsilon_s \exp[-2U^s(\phi, \varphi)](\dot{\Phi}^s)^2 - 2\mathcal{N}^{-2} V(\phi) \right\},$$

where $\dot{x} \equiv dx/du$,

$$V = V(\phi) = -w\Lambda e^{2\gamma_0(\phi)} + \frac{w}{2} \sum_{i=0}^{n} \xi_i d_i e^{-2\phi^i + 2\gamma_0(\phi)} \qquad (3.3)$$

is the potential with

$$\gamma_0(\phi) \equiv \sum_{i=0}^{n} d_i \phi^i, \qquad (3.4)$$

and

$$\mathcal{N} = \exp(\gamma_0 - \gamma) > 0 \qquad (3.5)$$

is the lapse function,

$$U^s = U^s(\phi, \varphi) = -\chi_s \lambda_{a_s}(\varphi) + \sum_{i \in I_s} d_i \phi^i, \qquad (3.6)$$

$$\varepsilon_s = (-\varepsilon[g])^{(1-\chi_s)/2} \varepsilon(I_s) \theta_{a_s} \qquad (3.7)$$

for $s = (a_s, v_s, I_s) \in S$, $\varepsilon[g] = \text{sign} \det(g_{MN})$, (more explicitly (3.7) reads $\varepsilon_s = \varepsilon(I_s)\theta_{a_s}$ for $v_s = e$ and $\varepsilon_s = -\varepsilon[g]\varepsilon(I_s)\theta_{a_s}$, for $v_s = m$)

$$\chi_s = +1, \quad v_s = e; \qquad (3.8)$$

$$\chi_s = -1, \quad v_s = m, \qquad (3.9)$$

and

$$G_{ij} = d_i \delta_{ij} - d_i d_j \qquad (3.10)$$

are components of the "pure cosmological" minisupermetric; $i, j = 0, \ldots, n$ [52].

In the electric case ($\mathcal{F}^{(a,m,I)} = 0$) for finite internal space volumes V_i the action (3.2) coincides with the action (2.1) if $\mu_* = -w/\kappa_0^2$, $\kappa^2 = \kappa_0^2 V_0 \ldots V_n$.

Action (3.2) may be also written in the form

$$S_\sigma = \frac{\mu_*}{2} \int du \mathcal{N} \left\{ \mathcal{G}_{\hat{A}\hat{B}}(X) \dot{X}^{\hat{A}} \dot{X}^{\hat{B}} - 2\mathcal{N}^{-2} V(X) \right\}, \qquad (3.11)$$

where $X = (X^{\hat{A}}) = (\phi^i, \varphi^\alpha, \Phi^s) \in \mathbf{R}^N$, and minisupermetric

$$\mathcal{G} = \mathcal{G}_{\hat{A}\hat{B}}(X) dX^{\hat{A}} \otimes dX^{\hat{B}} \qquad (3.12)$$

on minisuperspace

$$\mathcal{M} = \mathbb{R}^N, \quad N = n + 1 + l + |S| \qquad (3.13)$$

($|S|$ is the number of elements in S) is defined by the relation

$$(\mathcal{G}_{\hat{A}\hat{B}}(X)) = \begin{pmatrix} G_{ij} & 0 & 0 \\ 0 & h_{\alpha\beta} & 0 \\ 0 & 0 & \varepsilon_s e^{-2U^s(X)} \delta_{ss'} \end{pmatrix}. \qquad (3.14)$$

The minisuperspace metric (3.12) may be also written in the form

$$\mathcal{G} = \bar{G} + \sum_{s \in S} \varepsilon_s \mathrm{e}^{-2U^s(x)} d\Phi^s \otimes d\Phi^s, \tag{3.15}$$

where $x = (x^A) = (\phi^i, \varphi^\alpha)$,

$$\bar{G} = \bar{G}_{AB} dx^A \otimes dx^B = G_{ij} d\phi^i \otimes d\phi^j + h_{\alpha\beta} d\varphi^\alpha \otimes d\varphi^\beta, \tag{3.16}$$

$$(\bar{G}_{AB}) = \begin{pmatrix} G_{ij} & 0 \\ 0 & h_{\alpha\beta} \end{pmatrix}, \tag{3.17}$$

$U^s(x) = U_A^s x^A$ is defined in (3.6) and

$$(U_A^s) = (d_i \delta_{iI_s}, -\chi_s \lambda_{a_s \alpha}). \tag{3.18}$$

Here

$$\delta_{iI} \equiv \sum_{j \in I} \delta_{ij} = \begin{array}{ll} 1, & i \in I \\ 0, & i \notin I \end{array} \tag{3.19}$$

is an indicator of i belonging to I. The potential (3.3) reads

$$V = (-w\Lambda)\mathrm{e}^{2U^\Lambda(x)} + \sum_{j=0}^{n} \frac{w}{2} \xi_j d_j \mathrm{e}^{2U^j(x)}, \tag{3.20}$$

where

$$U^j(x) = U_A^j x^A = -\phi^j + \gamma_0(\phi), \tag{3.21}$$

$$U^\Lambda(x) = U_A^\Lambda x^A = \gamma_0(\phi), \tag{3.22}$$

$$(U_A^j) = (-\delta_i^j + d_i, 0), \tag{3.23}$$

$$(U_A^\Lambda) = (d_i, 0). \tag{3.24}$$

The integrability of the Lagrange system (3.11) depends upon the scalar products of co-vectors U^Λ, U^j, U^s corresponding to $\bar{G}$:

$$(U, U') = \bar{G}^{AB} U_A U'_B, \tag{3.25}$$

where

$$(\bar{G}^{AB}) = \begin{pmatrix} G^{ij} & 0 \\ 0 & h^{\alpha\beta} \end{pmatrix} \tag{3.26}$$

is matrix inverse to (3.17). Here (as in [52])

$$G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D}, \tag{3.27}$$

$i, j = 0, \ldots, n$. These products have the following form

$$(U^i, U^j) = \frac{\delta_{ij}}{d_j} - 1, \tag{3.28}$$

$$(U^\Lambda, U^\Lambda) = -\frac{D - 1}{D - 2}, \tag{3.29}$$

$$(U^s, U^{s'}) = q(I_s, I_{s'}) + \chi_s \chi_{s'} \lambda_{a_s} \cdot \lambda_{a_{s'}}, \tag{3.30}$$

$$(U^s, U^i) = -\delta_{iI_s}, \tag{3.31}$$

where $s = (a_s, v_s, I_s)$, $s' = (a_{s'}, v_{s'}, I_{s'}) \in S$,

$$q(I, J) \equiv d(I \cap J) + \frac{d(I)d(J)}{2 - D}, \qquad (3.32)$$

$$\lambda_a \cdot \lambda_b \equiv \lambda_{a\alpha} \lambda_{b\beta} h^{\alpha\beta}. \qquad (3.33)$$

Relations (3.28)-(3.29) were found in [70] and (3.30) in [143].

4 Cosmological and spherically symmetric solutions

Here we put the following restrictions on the parameters of the model

$$\textbf{(i)} \qquad \Lambda = 0, \qquad (4.1)$$

i.e. the cosmological constant is zero,

$$\textbf{(ii)} \qquad \xi_0 \neq 0, \quad \xi_1 = \ldots = \xi_n = 0, \qquad (4.2)$$

i.e. one space is curved and others are Ricci-flat,

$$\textbf{(iii)} \qquad 0 \notin I_s, \quad \forall s = (a_s, v_s, I_s) \in S, \qquad (4.3)$$

i.e. all "brane" manifolds M_{I_s} (see (2.19)) do not contain M_0.

We also impose a block-orthogonality restriction on the set of vectors $(U^s, s \in S)$. Let

$$S = S_1 \sqcup \ldots \sqcup S_k, \qquad (4.4)$$

$S_i \neq \emptyset$, $i = 1, \ldots, k$, and

$$\textbf{(iv)} \; (U^s, U^{s'}) = d(I_s \cap I_{s'}) + \frac{d(I_s)d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{a_s\alpha} \lambda_{a_{s'}\beta} h^{\alpha\beta} = 0, \qquad (4.5)$$

for all $s = (a_s, v_s, I_s) \in S_i$, $s' = (a_{s'}, v_{s'}, I_{s'}) \in S_j$, $i \neq j$; $i, j = 1, \ldots, k$. Relation (4.4) means that the set S is a union of k non-intersecting (non-empty) subsets $S_1, \ldots, S_k$. According to (4.5) the set of vectors $(U^s, s \in S)$ has a block-orthogonal structure with respect to the scalar product (3.25), i.e. it splits into k mutually orthogonal blocks $(U^s, s \in S_i)$, $i = 1, \ldots, k$.

¿From **(i)**, **(ii)** we get for the potential (3.20)

$$V = \frac{1}{2} w \xi_0 d_0 e^{2U^0(x)}, \qquad (4.6)$$

where

$$(U^0, U^0) = \frac{1}{d_0} - 1 < 0 \qquad (4.7)$$

(see (3.28)).

¿From **(iii)** and (3.31) we get

$$(U^0, U^s) = 0 \tag{4.8}$$

for all $s \in S$. Thus, the set of co-vectors U^0, U^s, $s \in S$ (belonging to dual space $(\mathbb{R}^{n+1+l})^* \simeq \mathbb{R}^{n+1+l})$ has also a block-orthogonal structure with respect to the scalar product (3.25).

Here we fix the time gauge as follows

$$\gamma = \gamma_0, \quad \mathcal{N} = 1, \tag{4.9}$$

i.e the harmonic time gauge is used. Then we obtain the Lagrange system with the Lagrangian

$$L = \frac{\mu_*}{2} \mathcal{G}_{\hat{A}\hat{B}}(X) \dot{X}^{\hat{A}} \dot{X}^{\hat{B}} - \mu_* V \tag{4.10}$$

and the energy constraint

$$E = \frac{\mu_*}{2} \mathcal{G}_{\hat{A}\hat{B}}(X) \dot{X}^{\hat{A}} \dot{X}^{\hat{B}} + \mu_* V = 0. \tag{4.11}$$

Here we will integrate the Lagrange equations corresponding to the Lagrangian (4.10) with the energy-constraint (4.11) and hence we will find classical exact solutions when the restrictions (8.2), (8.3) from Appendix 1 are imposed.

The problem of integrability may be simplified if we integrate the Maxwell equations (for $s \in S_e$) and Bianchi identities (for $s \in S_m$):

$$\frac{d}{du} \left(\exp(-2U^s) \dot{\Phi}^s \right) = 0 \iff \dot{\Phi}^s = Q_s \exp(2U^s), \tag{4.12}$$

where Q_s are constants, $s \in S$.

Let

$$Q_s \neq 0, \tag{4.13}$$

for all $s \in S$.

For fixed $Q = (Q_s, s \in S)$ the Lagrange equations for the Lagrangian (4.10) corresponding to $(x^A) = (\phi^i, \varphi^\alpha)$, when equations (4.12) are substituted are equivalent to the Lagrange equations for the Lagrangian

$$L_Q = \frac{1}{2} \bar{G}_{AB} \dot{x}^A \dot{x}^B - V_Q, \tag{4.14}$$

where

$$V_Q = V + \frac{1}{2} \sum_{s \in S} \varepsilon_s Q_s^2 \exp[2U^s(x)], \tag{4.15}$$

$(\bar{G}_{AB})$ and V are defined in (3.17) and (4.6) respectively. The zero-energy constraint (4.11) reads

$$E_Q = \frac{1}{2} \bar{G}_{AB} \dot{x}^A \dot{x}^B + V_Q = 0. \tag{4.16}$$

When the conditions **(i)**–**(iv)** are satisfied exact solutions to Lagrange equations corresponding to (4.14) with the potential (4.15) and V from (4.6) could be readily obtained using the relations from Appendix 2.

The solutions read:

$$x^A(u) = -\frac{U^{0A}}{(U^0, U^0)} \ln|f_0(u)| - \sum_{s \in S} \eta_s \nu_s^2 U^{sA} \ln|f_s(u)| + c^A u + \bar{c}^A. \qquad (4.17)$$

Functions f_0 and f_s in (4.17) are the following:

$$f_0(u) = |\xi_0(d_0 - 1)|^{1/2} s(u - u_0, w\xi_0, C_0) = \qquad (4.18)$$

$$\left|\frac{\xi_0(d_0 - 1)}{C_0}\right|^{1/2} \sinh(\sqrt{C_0}(u - u_0)), \ C_0 > 0, \ \xi_0 w > 0; \qquad (4.19)$$

$$\left|\frac{\xi_0(d_0 - 1)}{C_0}\right|^{1/2} \sin(\sqrt{|C_0|}(u - u_0)), \ C_0 < 0, \ \xi_0 w > 0; \qquad (4.20)$$

$$\left|\frac{\xi_0(d_0 - 1)}{C_0}\right|^{1/2} \cosh(\sqrt{C_0}(u - u_0)), \ C_0 > 0, \ \xi_0 w < 0; \qquad (4.21)$$

$$|\xi_0(d_0 - 1)|^{1/2} (u - u_0), \ C_0 = 0, \ \xi_0 w > 0, \qquad (4.22)$$

and

$$f_s(u) = \frac{|Q_s|}{|\nu_s|} s(u - u_s, -\eta_s \varepsilon_s, C_s) = \qquad (4.23)$$

$$\frac{|Q_s|}{|\nu_s||C_s|^{1/2}} \sinh(\sqrt{C_s}(u - u_s)), \ C_s > 0, \ \eta_s \varepsilon_s < 0; \qquad (4.24)$$

$$\frac{|Q_s|}{|\nu_s||C_s|^{1/2}} \sin(\sqrt{|C_s|}(u - u_s)), \ C_s < 0, \ \eta_s \varepsilon_s < 0; \qquad (4.25)$$

$$\frac{|Q_s|}{|\nu_s||C_s|^{1/2}} \cosh(\sqrt{C_s}(u - u_s)), \ C_s > 0, \ \eta_s \varepsilon_s > 0; \qquad (4.26)$$

$$\frac{|Q^s|}{|\nu_s|}(u - u_s), \ C_s = 0, \ \eta_s \varepsilon_s < 0, \qquad (4.27)$$

where C_0, C_s, u_0, u_s are constants, $s \in S$. The function $s(u, \xi, C)$ is defined in Appendix 2.

The parameters $\eta_s = \pm 1$, $\nu_s \neq 0$, $s \in S$, satisfy the relations

$$\sum_{s' \in S} (U^s, U^{s'}) \eta_{s'} \nu_{s'}^2 = 1, \qquad (4.28)$$

for all $s \in S$, with scalar products $(U^s, U^{s'})$ defined in (3.30).

The constants C_s, u_s are coinciding inside blocks:

$$u_s = u_{s'}, \qquad C_s = C_{s'}, \qquad (4.29)$$

$s, s' \in S_i$, $i = 1, \ldots, k$ (see relation (9.13) from Appendix 2). The ratios $\varepsilon_s Q_s^2/(\eta_s \nu_s^2)$ are also coinsiding inside blocks, or, equivalently,

$$\varepsilon_s \eta_s = \varepsilon_{s'} \eta_{s'} \tag{4.30}$$

$$\frac{Q_s^2}{\nu_s^2} = \frac{Q_{s'}^2}{\nu_{s'}^2}, \tag{4.31}$$

$s, s' \in S_i$, $i = 1, \ldots, k$. Here we used the relations (4.7), (4.8).

The contravariant components $U^{rA} = \bar{G}^{AB} U_B^r$ are [166]

$$U^{0i} = -\frac{\delta_0^i}{d_0}, \quad U^{0\alpha} = 0, \tag{4.32}$$

$$U^{si} = G^{ij} U_j^s = \delta_{iI_s} - \frac{d(I_s)}{D-2}, \quad U^{s\alpha} = -\chi_s \lambda_{a_s}^\alpha. \tag{4.33}$$

Using (4.17), (4.7), (4.33) and (4.32) we obtain

$$\phi^i = \frac{\delta_0^i}{1-d_0} \ln|f_0| - \sum_{s \in S} \eta_s \nu_s^2 \left(\delta_{iI_s} - \frac{d(I_s)}{D-2} \right) \ln|f_s| + c^i u + \bar{c}^i, \tag{4.34}$$

and

$$\varphi^\alpha = \sum_{s \in S} \eta_s \nu_s^2 \chi_s \lambda_{a_s}^\alpha \ln|f_s| + c^\alpha u + \bar{c}^\alpha, \tag{4.35}$$

$\alpha = 1, \ldots, l$.

Vectors $c = (c^A)$ and $\bar{c} = (\bar{c}^A)$ satisfy the linear constraint relations (see (9.20) in Appendix 2)

$$U^0(c) = U_A^0 c^A = -c^0 + \sum_{j=0}^n d_j c^j = 0, \tag{4.36}$$

$$U^0(\bar{c}) = U_A^0 \bar{c}^A = -\bar{c}^0 + \sum_{j=0}^n d_j \bar{c}^j = 0, \tag{4.37}$$

$$U^s(c) = U_A^s c^A = \sum_{i \in I_s} d_i c^i - \chi_s \lambda_{a_s \alpha} c^\alpha = 0, \tag{4.38}$$

$$U^s(\bar{c}) = U_A^s \bar{c}^A = \sum_{i \in I_s} d_i \bar{c}^i - \chi_s \lambda_{a_s \alpha} \bar{c}^\alpha = 0, \tag{4.39}$$

$s \in S$. The (3.4) reads

$$\gamma_0(\phi) = \frac{d_0}{1-d_0} \ln|f_0| + \sum_{s \in S} \frac{d(I_s)}{D-2} \eta_s \nu_s^2 \ln|f_s| + c^0 u + \bar{c}^0. \tag{4.40}$$

The zero-energy constraint reads (see Appendix 2)

$$E = E_0 + \sum_{s \in S} E_s + \frac{1}{2} \bar{G}_{AB} c^A c^B = 0, \tag{4.41}$$

where $E_0 = C_0 (U^0, U^0)^{-1}/2$, $E_s = C_s(\eta_s \nu_s^2)/2$. Using relations (3.10), (3.17), (4.7) and (4.36) we rewrite (4.41) as

$$C_0 \frac{d_0}{d_0 - 1} = \sum_{s \in S} C_s \nu_s^2 \eta_s + h_{\alpha\beta} c^\alpha c^\beta + \sum_{i=1}^n d_i (c^i)^2 + \frac{1}{d_0 - 1} \left(\sum_{i=1}^n d_i c^i \right)^2 . \quad (4.42)$$

¿From relation

$$\exp(2U^s) = f_s^{-2}, \quad (4.43)$$

following from (4.5), (4.8), (4.17), (4.38) and (4.39) we get for electric-type forms (2.22)

$$\mathcal{F}^s = Q_s f_s^{-2} du \wedge \tau(I_s), \quad (4.44)$$

$s \in S_e$, and for magnetic-type forms (2.23)

$$\mathcal{F}^s = e^{-2\lambda_a(\varphi)} * \left[Q_s f_s^{-2} du \wedge \tau(I_s) \right] = \bar{Q}_s \tau(\bar{I}_s), \quad (4.45)$$

$s \in S_m$, where $\bar{Q}_s = Q_s \varepsilon(I_s) \mu(I_s) w$ and $\mu(I) = \pm 1$ is defined by the relation $\mu(I) du \wedge \tau(I_0) = \tau(\bar{I}) \wedge du \wedge \tau(I)$. The relation (4.45) follows from the formula (5.26) from [143] (for $\gamma = \gamma_0$).

Relations for the metric follows from (4.34) and (4.40)

$$g = \left(\prod_{s \in S} [f_s^2(u)]^{\eta_s d(I_s) \nu_s^2/(D-2)} \right) \left\{ [f_0^2(u)]^{d_0/(1-d_0)} e^{2c^0 u + 2\bar{c}^0} \quad (4.46) \right.$$

$$\times [wdu \otimes du + f_0^2(u)g^0] + \sum_{i=1}^n \left(\prod_{s \in S} [f_s^2(u)]^{-\eta_s \nu_s^2 \delta_{iI_s}} \right) e^{2c^i u + 2\bar{c}^i} g^i \right\}.$$

Thus, here we obtained the "block-orthogonal" generalization of the solution from [166]. This solution describes the evolution of $n + 1$ spaces $(M_0, g_0), \ldots, (M_n, g_n)$, where (M_0, g_0) is an Einstein space of non-zero curvature, and (M_i, g^i) are "internal" Ricci-flat spaces, $i = 1, \ldots, n$; in the presence of several scalar fields and forms. The solution is presented by relations (4.35), (4.44)-(4.46) with the functions f_0, f_s defined in (4.18)–(4.27) and the relations on the parameters of solutions c^A, $\bar{c}^A$ $(A = i, \alpha)$, C_0, C_s, u_s, Q_s , η_s, ν_s $(s \in S)$ imposed in (4.28)–(4.31), (4.36)–(4.39), (4.42), respectively.

This solution describes a set of charged (by forms) overlapping p-branes ($p_s = d(I_s) - 1$, $s \in S$) "living" on submanifolds (isomorphic to) M_{I_s} (2.19), where the sets I_s do not contain 0, i.e. all p-branes live in "internal" Ricci-flat spaces.

The solution is valid if the dimensions of p-branes and dilatonic coupling vector satisfy the relations (4.5). In "orthogonal" non-composite case these solutions were considered in [154, 153] (electric case) and [151] (electro-magnetic case). For $n = 1$ see also [156, 130]. In block-orthogonal (non-composite) case a special class of solutions with ν_s^2 coinciding inside blocks was considered earlier in [167].

5 Spherically symmetric and black hole solutions

Here we consider the spherically symmetric case

$$w = 1, \quad M_0 = S^{d_0}, \quad g^0 = d\Omega_{d_0}^2, \tag{5.1}$$

where $d\Omega_{d_0}^2$ is the canonical metric on a unit sphere S^{d_0}, $d \geq 2$. We also assume that

$$M_1 = \mathbb{R}, \quad g^1 = -dt \otimes dt \tag{5.2}$$

(here M_1 is a time manifold) and

$$1 \in I_s, \quad \forall s \in S, \tag{5.3}$$

i. e. all p-branes have a common time direction t.

For integration constants we put $\bar{c}^A = 0$,

$$c^A = \bar{\mu}(\bar{b}^A - b^A), \tag{5.4}$$

$$\bar{b}^A = \bar{\mu} \sum_{r \in \bar{S}} \eta_r \nu_r^2 U^{rA} - \bar{\mu}\delta_1^A, \tag{5.5}$$

$$C_0 = \bar{\mu}^2, \tag{5.6}$$

$$C_s = \bar{\mu}^2 b_s^2, \qquad b_s > 0, \tag{5.7}$$

where $\bar{\mu} > 0$, $\bar{S} = \{0\} \cup S$ and $\eta_0 \nu_0^2 = (U^0, U^0)^{-1}$.

The only essential restrictions imposed are the inequalities $C_0, C_s > 0$ that cut a subclass in the class of solutions from Section 4. This subclass contains non-extremal black hole solutions and its "Kasner-like" (non-black-hole) deformations. For extremal black hole solutions one should consider the special case $C_0 = C_s = 0$. (For extremal black hole solutions and its multicenter generalizations see [168].)

Due to (4.29) the parameters b_s, $s \in S$, are coinciding inside blocks:

$$b_s = b_{s'}, \tag{5.8}$$

$s, s' \in S_i$, $i = 1, \ldots, k$.

It may be verified that the restrictions (4.36) and (4.38) are satisfied identically if and only if

$$U^0(b) = U_A^0 b^A = -b^0 + \sum_{j=0}^{n} d_j b^j = 1, \tag{5.9}$$

$$U^s(b) = U_A^s b^A = \sum_{i \in I_s} d_i b^i - \chi_s \lambda_{a_s \alpha} b^\alpha = 1, \tag{5.10}$$

$s \in S$. This follows from identities $U^0(\bar{b}) = 1$ and $U^s(\bar{b}) = 1$, $s \in S$.

Relation (4.42) reads

$$\sum_{s \in S} \eta_s \nu_s^2 (b_s^2 - 1) + h_{\alpha\beta} b^\alpha b^\beta + \sum_{i=1}^n d_i (b^i)^2 + \frac{1}{d_0 - 1} \left(\sum_{i=1}^n d_i b^i \right)^2 = \frac{d_0}{d_0 - 1}, \quad (5.11)$$

where the relation (equivalent to (5.9))

$$b^0 = \frac{1}{1 - d_0} \left[\sum_{j=1}^n d_j b^j - 1 \right], \quad (5.12)$$

is used.

Now we rewrite a solution (under restrictions imposed) in a so-called "Kasner-like" form that is more suitable for analysing the behaviour at large distances and for singling out the black hole solutions. For this reason we introduce a new radial variable $R = R(u)$ by relations

$$\exp(-2\bar\mu u) = 1 - \frac{2\mu}{R^{\bar d}}, \quad \mu = \bar\mu / \bar d > 0, \quad \bar d = d_0 - 1, \quad (5.13)$$

$u > 0$, $R^{\bar d} > 2\mu$. For the function

$$f_s(u) = \frac{|Q_s|}{2\bar\mu b_s |\nu_s|} [\exp(\bar\mu b_s(u - u_s)) + \eta_s \varepsilon_s \exp(-\bar\mu b_s(u - u_s))] \quad (5.14)$$

we put the restriction $f_s(0) = 1$, or, equivalently,

$$\exp(-\bar\mu b_s u_s) + \eta_s \varepsilon_s \exp(\bar\mu b_s u_s) = \frac{2\bar\mu b_s |\nu_s|}{|Q_s|}. \quad (5.15)$$

This restriction guarantees the asymptotical flatness of the $(2 + d_0)$-dimensional section of the metric in the limit $R \to +\infty$ (or, when, $u \to +0$)). It follows from (5.15) that $u_s < 0$ for $\eta_s \varepsilon_s = -1$. In any case $f_s(u) > 0$ for $u \geq 0$.

Then, solutions for the metric and scalar fields (see (4.35), (4.46)) may be written as follows

$$g = \left(\prod_{s \in S} \bar H_s^{2\eta_s d(I_s)\nu_s^2/(D-2)} \right) \left\{ F^{b^0 - 1} dR \otimes dR + R^2 F^{b^0} d\Omega_{d_0}^2 \right. \quad (5.16)$$

$$\left. - \left(\prod_{s \in S} \bar H_s^{-2\eta_s \nu_s^2} \right) F^{b^1} dt \otimes dt + \sum_{i=2}^n \left(\prod_{s \in S} \bar H_s^{-2\eta_s \nu_s^2 \delta_{iI_s}} \right) F^{b^i} g^i \right\},$$

$$\varphi^\alpha = \sum_{s \in S} \eta_s \nu_s^2 \chi_s \lambda_{a_s}^\alpha \ln \bar H_s + \frac{1}{2} b^\alpha \ln F, \quad (5.17)$$

where

$$F = 1 - \frac{2\mu}{R^{\bar d}}, \quad (5.18)$$

$$\bar H_s = \hat H_s F^{(1 - b_s)/2}, \quad (5.19)$$

$$\hat H_s = 1 + \hat P_s \frac{(1 - F^{b_s})}{2\mu b_s}, \quad (5.20)$$

$$\hat{P}_s = -\varepsilon_s \eta_s P_s, \tag{5.21}$$

$$P_s = \frac{|Q_s|}{\bar{d}|\nu_s|} \exp(\mu u_s) > 0, \tag{5.22}$$

$s \in S$. Due to (4.29)-(4.31) parameters P_s and $\hat{P}_s$ are coinciding inside blocks:

$$P_s = P_{s'}, \qquad \hat{P}_s = \hat{P}_{s'}, \tag{5.23}$$

$s, s' \in S_i$, $i = 1, \dots, k$. Parameters b_s are also coinciding inside blocks, see (5.8). Parameters b_s, b^i, b^α obey the relations (5.10)-(5.12).

The fields of forms are given by (2.22), (2.23) with

$$\Phi^s = \frac{\nu_s}{H_s'}, \tag{5.24}$$

$$H_s' = \left[1 - P_s' \hat{H}_s^{-1} \frac{(1 - F^{b_s})}{2\mu b_s} \right]^{-1}, \tag{5.25}$$

$$P_s' = -\frac{Q_s}{\nu_s \bar{d}}. \tag{5.26}$$

$s \in S$. It follows from (5.15), (5.20), (5.21) and (5.26) that

$$(P_s')^2 = P_s(\hat{P}_s + 2b_s\mu) = -\varepsilon_s\eta_s\hat{P}_s(\hat{P}_s + 2b_s\mu), \tag{5.27}$$

$s \in S$. This relation is self-consistent, i.e. its left- and right-hand sides have the same sign, since due to (5.15) and (5.22)

$$P_s < 2\mu b_s \tag{5.28}$$

for $\varepsilon_s\eta_s = +1$ and hence

$$\hat{P}_s > -2b_s\mu, \tag{5.29}$$

for all $s \in S$.

5.1   Black hole solutions

Here we show that the black hole solution ¿from [168] may be obtained from our spherically-symmetric solutions (5.16)-(5.27) when

$$b^1 = b_s = 1, \qquad b^i = b^\alpha = 0, \tag{5.30}$$

$s \in S$, $i = 0, 2, \dots, n$, $\alpha = 1, \dots, l$.

Under relations (5.30) imposed the metric and scalar fields (5.16) and (5.17) read

$$g = \left(\prod_{s \in S} \hat{H}_s^{2\eta_s d(I_s)\nu_s^2/(D-2)} \right) \left\{ \frac{dR \otimes dR}{1 - 2\mu/R^{\bar{d}}} + R^2 d\Omega_{d_0}^2 \right. \tag{5.31}$$

$$\left. - \left(\prod_{s \in S} \hat{H}_s^{-2\eta_s\nu_s^2} \right) \left(1 - \frac{2\mu}{R^{\bar{d}}} \right) dt \otimes dt + \sum_{i=2}^n \left(\prod_{s \in S} \hat{H}_s^{-2\eta_s\nu_s^2\delta_{iI_s}} \right) g^i \right\},$$

$$\varphi^\alpha = \sum_{s \in S} \eta_s \nu_s^2 \chi_s \lambda_{a_s}^\alpha \ln \hat{H}_s, \tag{5.32}$$

where $\mu > 0$, $R^{\bar{d}} > 2\mu$ and

$$\hat{H}_s = 1 + \frac{\hat{P}_s}{R^{\bar{d}}}, \quad \hat{P}_s > -2\mu, \tag{5.33}$$

$\hat{P}_s \neq 0$, $s \in S$. Parameters $\hat{P}_s$ are coinciding inside blocks (see (5.23)).

The fields of forms are given by (2.22), (2.23) with

$$\Phi^s = \frac{\nu_s}{H_s'}, \tag{5.34}$$

$$H_s' = \left(1 - \frac{P_s'}{\hat{H}_s R^{\bar{d}}}\right)^{-1} = 1 + \frac{P_s'}{R^{\bar{d}} + \hat{P}_s - P_s'}, \tag{5.35}$$

$s \in S$. Here

$$(P_s')^2 = -\varepsilon_s \eta_s \hat{P}_s (\hat{P}_s + 2\mu), \tag{5.36}$$

and

$$\varepsilon_s \eta_s \hat{P}_s < 0, \tag{5.37}$$

$s \in S$. Parameters ν_s satisfy relations (4.28).

The solution obtained describes non-extremal charged p-brane black holes with block-orthogonal intersection rules. The exterior horizon corresponds to $R^{\bar{d}} \to 2\mu$.

Let

$$\varepsilon_s \eta_s = -1, \tag{5.38}$$

$s \in S$. This restriction is satisfied in orthogonal case, when pseudo-Euclidean p-branes in a space-time of pseudo-Euclidean signature are considered (in this case all $\varepsilon(I_s) = -1$, $\varepsilon[g] = -1$), all $\theta_s = +1$ in the action (2.1) and $\eta_s = \text{sign}(U^s, U^s) = +1$).

Under restrictions (5.38) imposed our solutions agree with the solutions with orthogonal intersection rules ¿from Refs. [131], [149], [150] ($d_1 = \ldots = d_n = 1$, $\eta_s = +1$), [151] ($\eta_s = +1$, non-composite case) and block-orthogonal ones from [167] (for ν_s coinciding inside blocks).

Hawking temperature. The Hawking temperature corresponding to the solution is (see also [151, 150])

$$T_H(\mu) = \frac{\bar{d}}{4\pi(2\mu)^{1/\bar{d}}} \prod_{s \in S} \left(\frac{2\mu}{2\mu + \hat{P}_s}\right)^{\eta_s \nu_s^2}. \tag{5.39}$$

For fixed $\hat{P}_s > 0$ ($\varepsilon_s \eta_s = -1$) and $\mu \to +0$ we get $T_H(\mu) \to 0$ for the extremal black hole configurations [168] satisfying

$$\xi = \sum_{s \in S} \eta_s \nu_s^2 - \bar{d}^{-1} > 0. \tag{5.40}$$

6 Post-Newtonian approximation

Let $d_0 = 2$. Here we consider the 4-dimensional section of the metric (5.16)

$$g^{(4)} = U \left\{ F^{b^0 - 1} dR \otimes dR + F^{b^0} R^2 d\hat{\Omega}_2^2 - U_1 F^{b^1} dt \otimes dt \right\}, \tag{6.1}$$

where $F = 1 - (2\mu/R)$, and

$$U = \prod_{s \in S} \bar{H}_s^{2\eta_s d(I_s)\nu_s^2/(D-2)}, \tag{6.2}$$

$$U_1 = \prod_{s \in S} \bar{H}_s^{-2\eta_s \nu_s^2}, \tag{6.3}$$

$R > 2\mu$.

We may suppose that some real astrophysical objects (e.g. stars) are described by the 4-dimensional "physical" metric (6.1), i.e. they are "traces" of extended multidimensional objects (charged p-branes).

Introducing a new radial variable ρ by the relation

$$R = \rho \left(1 + \frac{\mu}{2\rho} \right)^2, \tag{6.4}$$

$(\rho > \mu/2)$, we rewrite the metric (6.1) in a 3-dimensional conformally-flat form

$$g^{(4)} = U \left\{ -U_1 F^{b^1} dt \otimes dt + F^{b^0} \left(1 + \frac{\mu}{2\rho} \right)^4 \delta_{ij} dx^i \otimes dx^j \right\}, \tag{6.5}$$

$$F = \left(1 - \frac{\mu}{2\rho} \right)^2 \left(1 + \frac{\mu}{2\rho} \right)^{-2} \tag{6.6}$$

where $\rho^2 = |x|^2 = \delta_{ij} x^i x^j$ $(i, j = 1, 2, 3)$.

For possible physical applications we should calculate the post-Newtonian parameters β and γ (Eddington parameters) using the following relations (see, for example, [170] and references therein)

$$g_{00}^{(4)} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{6.7}$$

$$g_{ij}^{(4)} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{6.8}$$

$i, j = 1, 2, 3$, where

$$V = \frac{GM}{\rho} \tag{6.9}$$

is the Newton's potential, G is the gravitational constant, M is the gravitational mass. ¿From (6.5)-(6.9) we get

$$GM = \mu b^1 + \sum_{s \in S} \eta_s \nu_s^2 [\hat{P}_s + (b_s - 1)\mu] \left(1 - \frac{d(I_s)}{D-2} \right) \tag{6.10}$$

and for $GM \neq 0$

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} \eta_s \nu_s^2 \hat{P}_s (\hat{P}_s + 2b_s \mu) \left(1 - \frac{d(I_s)}{D-2}\right) \quad (6.11)$$

$$\gamma - 1 = -\frac{1}{GM}\left[\mu(b^0 + b^1 - 1) + \sum_{s \in S} \eta_s \nu_s^2 [\hat{P}_s + (b_s - 1)\mu]\left(1 - 2\frac{d(I_s)}{D-2}\right)\right] \quad (6.12)$$

It follows from (5.27), (6.11) and the inequalities $d(I_s) < D-2$ (for all $s \in S$) that the following inequalities take place

$$\beta > 1, \text{ if all } \varepsilon_s = -1, \quad (6.13)$$
$$\beta < 1, \text{ if all } \varepsilon_s = +1. \quad (6.14)$$

There exists a large variety of configurations with $\beta = 1$ when the relation $\varepsilon_s = $ const is broken.

There exist also non-trivial p-brane configurations with $\gamma = 1$.

Proposition. Let the set of p-branes consist of several pairs of electric and magnetic branes. Let any such pair $(s, \bar{s} \in S)$ correspond to the same colour index, i.e. $a_s = a_{\bar{s}}$, and $\hat{P}_s = \hat{P}_{\bar{s}}$, $b_s = b_{\bar{s}}$, $\eta_s \nu_s^2 = \eta_{\bar{s}} \nu_{\bar{s}}^2$. Then for $b^0 + b^1 = 1$ we get

$$\gamma = 1. \quad (6.15)$$

The Proposition can be readily proved using the relation $d(I_s) + d(I_{\bar{s}}) = D - 2$, following from (2.28) and (2.29).

Observational restrictions. The observations in the solar system give the tight constraints on the Eddington parameters [170]

$$\gamma = 1.000 \pm 0.002 \quad (6.16)$$
$$\beta = 0.9998 \pm 0.0006. \quad (6.17)$$

The first restriction is a result of the Viking time-delay experiment [171]. The second restriction follows from (6.16) and the analysis of the laser ranging data to the Moon. In this case a high precision test based on the calculation of the combination $(4\beta - \gamma - 3)$ appearing in the Nordtvedt effect [173] is used [172]. We note, that as it was pointed in [170] the "classic" tests of general relativity, i.e. the Mercury-perihelion and light deflection tests, are somewhat outdated.

For small enough $\hat{p}_s = \hat{P}_s/GM$, $b_s - 1$, $b^1 - 1$, b^i $(i > 1)$ of the same order we get $GM \sim \mu$ and hence

$$\beta - 1 \sim \sum_{s \in S} \eta_s \nu_s^2 \hat{p}_s \left(1 - \frac{d(I_s)}{D-2}\right) \quad (6.18)$$

$$\gamma - 1 \sim -b^0 - b^1 + 1 - \sum_{s \in S} \eta_s \nu_s^2 [\hat{p}_s + (b_s - 1)]\left(1 - 2\frac{d(I_s)}{D-2}\right), \quad (6.19)$$

i.e. $\beta - 1$ and $\gamma - 1$ are of the same order. Thus for small enough $\hat{p}_s$, $b_s - 1$, $b^1 - 1$, b^i $(i > 1)$ it is possible to fit the "solar system" restrictions (6.16) and (6.17).

There exists also another possibility to satisfy these restrictions.

One brane case. Let us consider a special case of one p-brane. In this case we have

$$\eta_s \nu_s^{-2} = d(I_s) \left(1 - \frac{d(I_s)}{D - 2} \right) + \lambda^2. \tag{6.20}$$

Relations (6.11), (6.12) and (6.20) imply that for large enough values of (dilatonic coupling constant squared) λ^2 and $b^0 + b^1 = 1$ it is possible to perform the "fine tuning" the parameters (β, γ) near the point $(1, 1)$ even if the parameters $\hat{P}_s$ are big.

7 Conclusions

In this paper we obtained exact solutions to Einstein equations for the multi-dimensional cosmological model describing the evolution of n Ricci-flat spaces and one Einstein space M_0 of non-zero curvature in the presence of composite electro-magnetic p-branes. The solutions were obtained in the block-orthogonal case (4.5), when p-branes do not "live" in M_0. We also considered the spherically-symmetric solutions containing non-extremal p-brane black holes [167, 168]. The relations for post-Newtonian parameters β and γ are obtained.

Appendix 1: Restrictions on p-brane configurations

Restrictions on $\Omega_{a,v}$ [166]. Let

$$w_1 \equiv \{i \mid i \in \{0, \ldots, n\}, \ d_i = 1\}. \tag{8.1}$$

The set w_1 describes all 1-dimensional manifolds among M_i $(i \geq 0)$. We impose the following restrictions on the sets $\Omega_{a,v}$ (2.24):

$$W_{ij}(\Omega_{a,v}) = \emptyset, \tag{8.2}$$

$a \in \Delta$; $v = e, m$; $i, j \in w_1$, $i < j$ and

$$W_j^{(1)}(\Omega_{a,m}, \Omega_{a,e}) = \emptyset, \tag{8.3}$$

$a \in \Delta$; $j \in w_1$. Here

$$W_{ij}(\Omega_*) \equiv \{(I, J) \mid I, J \in \Omega_*, \ I = \{i\} \sqcup (I \cap J), \ J = \{j\} \sqcup (I \cap J)\}, \tag{8.4}$$

$i, j \in w_1$, $i \neq j$, $\Omega_* \subset \Omega_0$ and

$$W_j^{(1)}(\Omega_{a,m}, \Omega_{a,e}) \equiv \{(I, J) \in \Omega_{a,m} \times \Omega_{a,e} | \bar{I} = \{j\} \sqcup J\}, \qquad (8.5)$$

$j \in w_1$. In (8.5)
$$\bar{I} \equiv I_0 \setminus I \qquad (8.6)$$

is "dual" set, $(I_0 = \{0, 1, \ldots, n\})$.

The restrictions (8.2) and (8.3) are trivially satisfied when $n_1 \leq 1$ and $n_1 = 0$ respectively, where $n_1 = |w_1|$ is the number of 1-dimensional manifolds among M_i. They are also satisfied in the non-composite case when all $|\Omega_{a,v}| = 1$. For $n_1 \geq 2$ and $n_1 \geq 1$, respectively, these restrictions forbid certain pairs of two p-branes, corresponding to the same form $F^a, a \in \Delta$:

Appendix 2: Solutions with block-orthogonal set of vectors

Let

$$L = \frac{1}{2} < \dot{x}, \dot{x} > - \sum_{s \in S} A_s \exp(2 < b_s, x >) \qquad (9.1)$$

be a Lagrangian, defined on $V \times V$, where V is a n-dimensional vector space over $\mathbb{R}$, $A_s \neq 0$, $s \in S$; $S \neq \emptyset$, and $< \cdot, \cdot >$ is a non-degenerate real-valued quadratic form on V. Let

$$S = S_1 \sqcup \ldots \sqcup S_k, \qquad (9.2)$$

all $S_i \neq \emptyset$, and

$$< b_s, b_{s'} > = 0, \qquad (9.3)$$

for all $s \in S_i$, $s' \in S_j$, $i \neq j$; $i, j = 1, \ldots, k$.

Let us suppose that there exists a set $h_s \in \mathbb{R}$, $h_s \neq 0$, $s \in S$, such that

$$\sum_{s \in S} < b_s, b_{s'} > h_{s'} = -1, \qquad (9.4)$$

for all $s \in S$, and

$$\frac{A_s}{h_s} = \frac{A_{s'}}{h_{s'}}, \qquad (9.5)$$

$s, s' \in S_i$, $i = 1, \ldots, k$, (the ratio A_s/h_s is constant inside S_i).

Then, the Euler-Lagrange equations for the Lagrangian (9.1)

$$\ddot{x} + \sum_{s \in S} 2 A_s b_s \exp(2 < b_s, x >) = 0, \qquad (9.6)$$

have the following special solutions

$$x(t) = \frac{1}{2} \sum_{s \in S} h_s b_s \ln \left[y_s^2(t) \left| \frac{2A_s}{h_s} \right| \right] + \alpha t + \beta,$$ (9.7)

where $\alpha, \beta \in V$,

$$< \alpha, b_s > = < \beta, b_s > = 0,$$ (9.8)

$s \in S$, and functions $y_s(t) \neq 0$ satisfy the equations

$$\frac{d}{dt} \left(y_s^{-1} \frac{dy_s}{dt} \right) = -\xi_s y_s^{-2},$$ (9.9)

with

$$\xi_s = \text{sign} \left(\frac{A_s}{h_s} \right),$$ (9.10)

$s \in S$, and coincide inside blocks:

$$y_s(t) = y_{s'}(t),$$ (9.11)

$s, s' \in S_i$, $i = 1, \ldots, k$. More explicitly

$$y_s(t) = s(t - t_s, \xi_s, C_s),$$ (9.12)

where constants $t_s, C_s \in \mathbb{R}$ coincide inside blocks

$$t_s = t_{s'}, \qquad C_s = C_{s'},$$ (9.13)

$s, s' \in S_i$, $i = 1, \ldots, k$, and

$$s(t, \xi, C) \equiv \frac{1}{\sqrt{C}} \sinh(t\sqrt{C}), \quad \xi = +1, \quad C > 0;$$ (9.14)

$$\frac{1}{\sqrt{-C}} \sin(t\sqrt{-C}), \quad \xi = +1, \quad C < 0;$$ (9.15)

$$t, \quad \xi = +1, \quad C = 0;$$ (9.16)

$$\frac{1}{\sqrt{C}} \cosh(t\sqrt{C}), \quad \xi = -1, \quad C > 0.$$ (9.17)

For the energy

$$E = \frac{1}{2} < \dot{x}, \dot{x} > + \sum_{s \in S} A_s \exp(2 < b_s, x >)$$ (9.18)

corresponding to the solution (9.7) we have

$$E = \frac{1}{2} \sum_{s \in S} C_s(-h_s) + \frac{1}{2} < \alpha, \alpha > .$$ (9.19)

For dual vectors $u^s \in V^*$ defined as $u^s(x) = < b_s, x >$, $\forall x \in V$, we have $< u^s, u^l >_* = < b_s, b_l >$, where $< \cdot, \cdot >_*$ is dual form on V^*. The orthogonality conditions (9.8) read

$$u^s(\alpha) = u^s(\beta) = 0,$$ (9.20)

$s \in S$.

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